Effect of geometrical and material imperfections on damping flexural vibrations in plates with attached wedges of power law profile

Effect of geometrical and material imperfections on damping flexural vibrations in plates with attached wedges of power law profile

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Abstract

In the present paper, an efficient method of damping structural vibrations using the acoustic black hole effect is further investigated experimentally. This method is based on some specific properties of flexural wave propagation in tapered plates (wedges) of power-law profile that have to be partially covered by narrow thin strips of absorbing layers. Ideally,

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if the power-law exponent of the profile is equal or larger than two, the flexural wave never reaches the sharp edge and therefore never reflects back, which constitutes the acoustic black hole effect. It has been previously established theoretically and confirmed experimentally that this method of damping structural vibrations is very efficient even in the presence of edge truncations. The present work describes the results of the experimental studies of the effects of manufacturing intolerances on damping flexural vibrations in wedge-like structures of power-law profile. In particular, the effect of mechanical damage resulting from the use of cutting tools to wedge tips is investigated, including tip curling and early truncation, as well as the placement of absorbing layers on different wedge surfaces. Also, the effects of welded and glued bonding of wedge attachments to basic rectangular plates (strips) are investigated. The results show that, although the above-mentioned geometrical and material imperfections reduce the damping efficiency by varying degrees, the method of damping structural vibrations using the acoustic black hole effect is robust enough and can be used widely without the need of high precision manufacturing.

Keywords: Vibration damping; Acoustic black hole effect; Wedges of power-law profile; Geometrical and material imperfections.

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1 Introduction

Damping of unwanted structural vibrations remains a major issue in many branches of engineering, particularly in transport engineering. There are many methods of damping structural vibrations, the most common of which being attaching absorbing layers to the

surfaces of vibrating structures resulting in increase of structural wave attenuation [1-4]. An alternative way of damping applicable to finite structures is to reduce reflections of structural waves from their free edges and thus to reduce the amplitudes of structural resonances [1, 2].

An efficient method of reducing edge reflections of flexural waves from free edges of plates or bars has been recently considered theoretically [5, 6] and confirmed experimentally [7]. The method described in [5-7] utilises a gradual change in local thickness of a plate or a bar, also partly covered by a thin damping layer, from the value corresponding to the thickness of the basic plate or bar to almost zero (see Fig. 1). If the local thickness $h(x)$ of a plate or a bar is described by a power-law relationship: $h(x) = \varepsilon x^m$, where x is the distance from the edge along the median plane and ε and m are positive numbers, then flexural waves propagating towards the sharp edge slow down and grow in amplitude. According to the geometrical acoustics approximation used in [5, 6], for $m \geq 2$ the waves never reach the edge and thus become trapped, never reflecting back either. Therefore, it can be said that sharp edges of such power-law structures materialise acoustic 'black holes' for flexural waves.

Note that the above-mentioned 'unusual' effect of wedges of power-low profile in respect of their possible applications as vibration dampers has been first described in [8]. However, because of the inherent intolerances present in the production process, the physical or manufactured wedges do not have perfect ideal power-law shapes, largely due to ever-present truncations at their edges. Therefore, real reflection coefficients are high [8] and such structures alone can not be used as practical vibration dampers. The reflection coefficients, however, can be drastically reduced by depositing the above-mentioned thin strips of absorbing layers on surfaces of the wedges, very close to their sharp edges [5-7]. Thus, the combined effect of power-law geometry and of thin absorbing layers results in very efficient damping systems for flexural vibrations.

One of the advantages of the above-mentioned acoustic black holes as dampers of structural vibrations is that they are effective even for relatively thin and narrow strips of attached absorbing layers. Among their potential practical applications one can mention damping of excessive resonant vibrations in turbine and fan blades [7, 9] and damping of impact-induced vibrations in tennis racquets and golf clubs [9]. They can be used also for vibration damping in numerous engineering structures containing different types of plates and bars. In addition to the above-mentioned tapered plates and bars representing one-dimensional acoustic black holes, one can also use pits of power-law profile, or two-dimensional black holes that open new possibilities for applications. In particular, they can be made inside plate components of different aerospace structures, not compromising their overall rigidity [9]. In the same time, such pits combined with patches of thin absorbing layers can reduce quite substantially the levels of unwanted structural vibrations.

Subsequent theoretical and experimental investigations have confirmed the advantages of using the acoustic black hole effect for damping structural vibrations. In particular, the efficient damping of flexural vibrations of finite beams with tapered edges of power-law profile has been observed in [10]. The vibration damping effect of a tapered hole of power-law profile placed in one of the foci of an elliptical plate has been studied in [11, 12] using a scanning laser vibrometer. It has been shown in these investigations that if a shaker is applied to another focus then flexural waves are focused at the tapered hole, and the damping effect is strongly amplified. This was also confirmed using a one-dimensional numerical model [12]. In the papers [13-15], the numerical modelling and experimental measurements of point mobilities in rectangular plates with attached power-law wedges and in circular plates with central tapered holes have been carried out. The results show that substantial damping of resonant peaks takes place in rectangular plates with attached wedges, whereas in circular

plates with central tapered holes damping is not as large, which was expected since there was no focusing in this case.

Obviously, the geometry and material properties of wedges of power-law profile are very important for damping systems utilising the acoustic black hole effect. However, there were no investigations so far in which the influence of geometrical or material imperfections of power-law wedges on damping structural vibrations has been examined. The aim of the present paper is to undertake experimental investigation of the effects of geometrical and material imperfections on vibration damping in structures containing elastic wedges of power-law profile.

Initially, the effect of mechanical damage to wedge tips resulting from the use of cutting tools will be investigated, including tip curling and early truncation. Also, the placement of absorbing layers on different wedge surfaces will be studied. Then, the effects of welded and glued bonding of wedge attachments to basic rectangular plates (strips) will be investigated. Finally, the conclusions will be drawn regarding the effects of material and geometrical imperfections on damping structural vibrations.

2 Theoretical background

To understand the basic principle of the acoustic black hole effect it is instructive to consider the simplest case of a plane flexural wave propagating in the normal direction towards the edge of a free wedge described by a power-law relationship $h(x)= \varepsilon x^m$, where m is a positive rational number and ε is a positive constant. Since flexural wave propagation in such wedges can be described in the geometrical acoustics approximation (see [5, 6] for more detail), the integrated wave phase Φ resulting from the wave propagation from an arbitrary

point x on the wedge medium plane to the wedge tip ($x = 0$), which defines the wave solution in the geometrical acoustics approximation, can be written in the form:

$$\Phi = \int_0^x k(x)dx\,. \tag{1}$$

Here $k(x)$ is a local wavenumber of a flexural wave for a wedge in contact with vacuum: $k(x) = 12^{1/4}\, k_p^{1/2}(\varepsilon x^m)^{-1/2}$, where $k_p = \omega/c_p$ is the wavenumber of a symmetrical plate wave, $c_p = 2c_t(1-c_t^2/c_l^2)^{1/2}$ is its phase velocity, and c_l and c_t are longitudinal and shear wave velocities in a wedge material, and $\omega = 2\pi f$ is circular frequency. It can be seen that the integral in equation (1) diverges for $m \geq 2$. This means that the integrated phase Φ becomes infinite under these circumstances and the wave never reaches the edge. Therefore, it never reflects back either, i.e. the corresponding reflection coefficient R_0 is zero and the wave becomes trapped, thus indicating that the above mentioned ideal wedges can be considered as 'acoustic black holes' for incident flexural waves.

Real fabricated wedges, however, always have truncated edges. And this adversely affects their performance as acoustic black holes. If for ideal wedges of power-law profile (with $m \geq 2$) it follows from equation (1) that even an infinitely small material attenuation, described by an imaginary part of $k(x)$, would be sufficient for the total wave energy to be absorbed, this is not so for truncated wedges. Indeed, for truncated wedges the lower integration limit in equation (1) must be changed from 0 to a certain value x_0 describing the length of truncation. This implies that for typical values of attenuation in wedge materials, even very small truncations x_0 result in values of R_0 being as large as 50-70 % [5, 6], which destroys almost completely the expected acoustic black hole effect.

To radically improve the situation for real manufactured wedges (with truncations), it has been proposed to use covering the wedge surfaces by thin damping layers (films) [5-7], e.g. by polymeric films. Using the approach based on the geometrical acoustics approximation (see [5, 6] for more detail), the corresponding analytical expressions have been derived for the reflection coefficients of flexural waves from the edges of truncated wedges covered by thin absorbing layers. For example, for a truncated wedge of quadratic shape, $h(x) = \varepsilon x^2$, covered by a thin damping layer on one surface only, the following analytical expression has been derived for the resulting reflection coefficient R_0 at arbitrary distance x from the centre of the coordinate system $x = 0$ associated with the tip of an ideal (non-truncated) wedge [5, 6]:

$$R_0 = exp(-2\mu_1 - 2\mu_2). \tag{2}$$

Here

$$\mu_1 = \frac{12^{1/4} k_p^{\ 1/2} \eta}{4\varepsilon^{1/2}} \ln\left(\frac{x}{x_0}\right), \tag{3}$$

and

$$\mu_2 = \frac{3 \cdot 12^{1/4} k_p^{\ 1/2} \nu\delta}{8\varepsilon^{3/2}} \frac{E_2}{E_1} \frac{1}{x_0^{\ 2}} \left(1 - \frac{x_0^{\ 2}}{x^2}\right), \tag{4}$$

where x_0 is the length of the truncation, η and ν are the energy loss factors for the wedge and film materials respectively, δ is the film thickness, and E_1 and E_2 are Young's moduli of the wedge and film materials; other notations have been explained earlier.

Note that the expressions (2)-(4) for reflection coefficients of flexural waves are valid if the thickness of absorbing layers (films) δ is much smaller that the local thickness of the main wedge. Therefore, although being very useful and simple, these expressions can only be applicable either to wedges covered by very thin absorbing layers (thin films) or to wedges with large values of truncation x_0.

To extend the analysis to smaller values of wedge truncation and/or to thicker damping films one has to consider flexural wave propagation in wedges covered by damping layers of arbitrary thickness. Such an analysis has been undertaken in Ref. [6] using a more general approach to the description of the effect of damping layers on complex flexural rigidity of a sandwich plate. Using this approach, a more general expression for the reflection coefficient R_0 of a quadratic wedge covered by an absorbing layer on one side has been derived, which is not reproduced here for shortness. Although this expression has been derived under a number of simplifying assumptions, it is rather cumbersome and contains the integration that should be carried out numerically.

Numerical calculations according to the above-mentioned simple formulae (2)-(4) show that for typical parameters of metal wedges and polymeric damping films the values of the reflection coefficient R_0 at frequencies around 10 kHz can be as low as 3-6 %. Thus, in the presence of a damping film the values of the reflection coefficient from the tip of a truncated wedge of power-law profile are usually much smaller than those for a wedge with the same value of truncation x_0, but without a film. Obviously, it is both the specific geometrical properties of a power-law-shaped wedge in respect of flexural wave propagation and the effect of thin damping layers that result in such a significant reduction in the reflection coefficient. Note that almost all absorption of the incident wave energy takes place in the vicinity of the sharp edge of a wedge.

3 Manufacturing of experimental wedges

As it must be clear from the previous section, the manufacturing process is of great importance for practical implementation of wedges of power law profile as one could anticipate that a high level of precision is required in the formation of the profile to ensure maximum damping. Experimental samples in the present work were manufactured from 5 mm thick hot drawn mild steel sheets, which are more resistant to mechanical stresses incurred in the manufacturing process than cold drawn steels. Resilience to mechanical stresses results in fewer internal defects that could lead to increased elastic wave scattering. Plate and damping layer material properties are listed in Table 1, whereas sample dimensions are given in Table 2.

Wedges of power law profile, with $m = 2.2$, were milled at one end of the plates (strips). The milling machine was accurate to 0.01 mm, and the cutter head was inclined to reduce machining stresses. The cutter rotated at speed of 600 rpm in a manner that resulted in a pushing cut. This safeguarded the leading edge, thus reducing the damage. The main problem incurred by this method of manufacture was in obtaining a thin cut at the end of the wedge. To produce the correct power-law profile the plate was cut extremely thin, and as a result, it was susceptible to curling and damage to the tip of the profile due to heat and machining stresses (Fig. 2).

For the production of welded and glued samples (see Fig. 3), the following order of manufacture was used. The metal sheet was first cut into strips. The glued and homogeneous samples were profiled before the glued wedges were cut off. In the case of welded samples, to avoid damage to the profile due to the heat from welding, the un-profiled ends were cut and welded before profiling. A mild steel filler was used for both welds (Fig. 4), and the excess

weld material was removed from the top and bottom of the strip to minimise weight gain and ensure similar strip shape. This was to ensure that the welded samples remained as similar as possible to the reference strip.

Two types of weld have been used (see Fig. 4): (a) - Tungsten inert gas (TIG) weld (this method applies a separate filler rod and welding torch, and it is used for straight edge truncation welds); (b) – Metal inert gas (MIG) weld (in this method the filler rod is incorporated into the welding torch and generally requires a slanted cut in the materials to bond them together).

4 Experimental set-up

The experimental set-up was designed to allow almost free vibration of the sample plates, take the weight of the plate and introduce least damping to the system. A string suspension arrangement (Fig. 5) was chosen to support the test samples as it provides the least damping effect and nearly free boundary conditions, while still maintaining adequate strength to support the weight of the test sample.

An electromagnetic shaker provided the excitation input to the centre of the plate via a force transducer (Bruel and Kjaer Type 8200) attached to the surface of the plate via wax. Note that attachment via glue was tested as well. It was shown that below 3.8 kHz there was almost no difference in responses between the two attachment methods, whereas after this point the glue attachment resulted in increasingly greater damped response, particularly above 7 kHz where an increased damping of up to 5 dB could be seen. Therefore, wax was used as the attachment method for all the experiments of this paper.

The response was recorded by a broadband accelerometer (Bruel & Kjaer Type 4371) that was attached to the upper surface of the test sample, directly above the force transducer, also via wax (see Fig. 6). A broadband white noise signal was generated by the analyser and transferred to the test piece via a shaker. A Bruel & Kjaer Type 2035 analyser and amplifier were used to acquire the results. A schematic view of the experimental set-up is shown in Fig. 7. Point accelerance was measured in each test case. A measurement range of 0.015 - 9 kHz was used.

5 Results and discussion

5.1 Effect of an attached wedge of power-law profile on damping flexural vibrations

Two sets of initial measurements are described in this section: the effect of profiling a wedge with an added damping layer at the edge of a homogeneous narrow plate (strip), when compared to a reference strip, and the effect of adding a damping layer to a homogeneous wedge, when compared to an uncovered wedge.

Figure 8 shows a measured accelerance for a homogeneous profiled sample (m = 2.2) with an additional damping layer, as compared to the reference strip of constant thickness. It can be seen that there is little wedge-induced damping in the low frequency range, below 1.5 kHz. Then, there is a noticeable reduction (of about 6 dB) at 1.50 kHz. A smaller reduction of 4.5 and 2.5 dB compared to the reference peak is then recorded at 3.7 and 4.6 kHz respectively. The resonant peaks after this show a growing reduction with increasing frequency, ranging from 5.5 - 8.0 dB between 5.5 and 7.2 kHz. Finally, there is a slightly reduced damping effect shown at 7.2 kHz, where the reduction in peak amplitude is about 6

dB, with the greatest reduction of 8 dB occurring at 5.8 kHz. Due to the structural changes incurred when a wedge is profiled at one edge of a strip, a shift in resonant peaks is expected and can clearly be seen in Fig. 8. Note that the damping effect of a wedge of power-law profile has been documented earlier [5-7]. The present measurements extend these earlier results to the case of a wedge added to a strip.

Figure 9 shows measurement results for a homogeneous sample (m = 2.2) with early truncation and no wedge tip damage, with and without an additional damping layer. It can be seen that generally there is a small reduction in peak amplitudes for a wedge with added damping layer, increasing with frequency and ranging between 1.6 and 5 dB. These results confirm that the attachment of a damping layer to the wedge tip results in reduction in resonant peak amplitudes in comparison with a free wedge. The observed reductions though are much smaller than those observed in [7], which is likely to be caused by the large value of edge truncation in the present sample.

5.2 Effect of tip damage and early truncation of a wedge

This section describes the effect of tip damage in a wedge of the maximum possible (extended) length of 49 mm on its performance when compared to the same wedge when it has been cut to a reduced length of 39 mm, i.e. with a premature truncation.

Figure 10 shows the values of measured accelerance for a TIG welded sample (extended sample) of power-law profile (m = 2.2) compared to the TIG welded truncated sample. It can be seen that, in spite of the wedge tip damage, the resonant peaks of the extended sample show an increased amplitude reduction with increasing frequency. It ranges from 8.5 to 12.5 dB between 3.8 and 7.8 kHz, with a larger reduction of 15 dB recorded at around 8 kHz. This agrees with the predictions of [5, 6] (see also equations (2)-(4)) that the reflections from

truncated wedge tips increase with the increased truncation, thus resulting in poor damping characteristics in samples containing truncated power-law wedges.

The same measurements were performed with the glued bond (not shown here for brevity). The results followed a similar trend as in the case of the TIG welded sample, with the extended sample consistently performing better than the truncated sample. The maximum reduction of 7 dB occurred at 7.2 kHz in this case. The MIG welded sample and the homogeneous sample followed the same damping trend again, but with smaller increases in damping performance (not shown). The extended MIG welded sample achieved the maximum increase of 5.5 dB compared to the truncated sample and the extended homogeneous sample showed a 4.5 dB reduction over the truncated sample.

Despite the damage to the extended wedge tip, the increased length and resultant decrease in tip thickness provided the most efficient damping, thus demonstrating that the longer and thinner the wedge tip the greater the contribution of the acoustic black hole effect into the overall vibration damping.

5.3 Effect of the damping layer placement

According to the geometrical acoustics theory of the acoustic black hole effect [5, 6], the position of the damping layer in relation to which surface it is attached to should make no difference to the level of damping achieved. However the measured results shown below seem to contradict this statement when real manufactured wedges are considered.

The damping layer has been placed on the flat underside of the wedge tip and also on the profile side, as can be seen in Fig. 11. The results are shown in Fig. 12. It can be seen that a greater damping effect occurs when the damping layer is placed on the underside flat surface

of the wedge, the difference in the damping effect being quite noticeable. Even at low frequencies there is a difference between the two.

In order to obtain some statistical backing for this observation, the above measurements have been repeated for different pieces of damping tape taken from the same roll. The results showing the difference in the damping associated with placement of different pieces of damping tape on flat and curved surfaces can be seen in Table 3 for several peak frequencies. As it can be seen, the difference in the damping effect is preserved.

Possible reasons for this difference lie in the differences between the theoretical assumptions and the practicality of producing a wedge to meet the theoretical standards. The most likely reason is that due to the stepped nature of the top of the profile the damping layer does not make full contact with the entire surface beneath the strip, thus resulting in the reduced damping effect. Following from these results, the damping layers in all subsequent experiments of the present work have been placed on the flat underside of the wedge profile to attain the maximum damping possible.

5.4 Effect of bonding of a wedge to a strip

The present section describes the effects of bonding wedges to a constant thickness strip (Fig. 3). The wedges were bonded to a strip using two different types of weld, TIG and MIG (see Fig. 4), and also by glue. Each of these structures is compared to a reference strip and to a homogeneous structure that contains a wedge.

Figure 13 shows measured values of accelerance for a sample with the TIG-welded power-law profile (m = 2.2) in comparison with the results for the reference plate. It can be seen that there is a significant reduction in resonant peaks with increasing frequency, up to 8.0 - 10 dB between 5.5-9 kHz, with the greatest reduction of about 10 dB occurring at 7.2 kHz.

Figure 14 presents the measured accelerance for a sample with the MIG-welded power-law profile (m = 2.2) compared to the reference plate. Again, there is little damping in the low frequency range, below 1.5 kHz (a slight increase can in fact be seen at 0.29 and 0.81 kHz). A reduction of 6 dB compared to the reference peak is recorded at 3.7 kHz, and there is no change to the peak at 4.60 kHz. The resonant peaks after this show an increasing reduction with increasing frequency ranging from 6.5 to 7.5 dB between 5.5-9 kHz, with the greatest reduction of 7.5 dB occurring at 5.5 kHz.

Figure 15 shows the accelerance for a sample with the glued wedge of power-law profile (m = 2.2) compared to the reference plate. There is little to no damping in the low frequency range, below 1.50 kHz, except for a reduction of about 3 dB in the amplitude of the reference resonant peak located at 0.81 kHz. This reduction increases to 7.5 dB at 1.50 kHz. The resonant peaks then show an increased reduction with increasing frequency ranging from 8.5 to 10 dB between 4.6-9 kHz.

In order to account for statistical deviations in the effect of imperfections in the joining method used, the glued sample bond was broken, cleaned and reattached again. This procedure has been repeated five times. The results for the damping at several peak frequencies (compared to the reference plate) are shown in Table 4. As can be seen, in all five cases the amplitudes of the resonant peaks altered by no more than 2dB. Any discrepancies can be put down to the application method of the bonding agent and that it was not always possible to ensure an identical amount was added in each case. The trend however is consistent.

Finally, Fig. 16 shows the measured results for a homogeneous sample with power-law profile (m = 2.2) compared to the TIG-welded sample. There is little difference in damping performance in the frequency range below 4.5 kHz. After 4.5 kHz, the TIG-welded sample

consistently performs better than the homogeneous sample, with the reductions in comparison with the homogeneous sample by 4.5-6.0 dB.

It can be seen that the MIG-welded wedge performed the worst out of the three bonded samples, with a maximum damping of only 7.5 dB occurring at 5.5 kHz. The glued wedge performs better than the TIG-welded wedge at frequencies below 3.7 kHz, while the TIG-welded wedge performs better than the glued wedge over the remaining frequency range 3.7 – 9 kHz. Out of the three bonds tested, the TIG-welded wedge proved to yield the most significant reductions in comparison with the amplitudes of the reference strip resonance peaks over the largest range. The greatest reduction seen across the three samples was 10 dB occurring at 7.2 kHz, this reduction was seen in both the TIG and glued samples.

As expected, neither of the welds matches the damping performance of the homogeneous sample, however the TIG-welded sample above 4.5 kHz unexpectedly performs better than the homogeneous sample. The glued wedge also has a better damping performance than the homogeneous sample after this point.

The above differences may be due to flexural wave reflections from the material boundaries. For the weld used in this investigation, a mild steel filler was used for mild steel sheet metal, however the properties of the two metals were not identical, and the difference could increase after heating. Obviously, higher frequency waves could be affected more by the break in the plate and subsequent bonding. Note that the thermal cycles endured by the base metal during the welding process tend to alter the structure and properties of the metal considerably [16]. The most prominent defect resulting from heat in the welding process is the formation of a Martensite polycrystalline structure that could result in increased wave scattering and reflections on either side of the weld. Note that this effect can be reduced by preheating the steel before welding.

Although these increased reflections as a result of Martensite formation hinder waves from propagating into the wedge, it is possible that to some extent the waves that enter the wedge and are reflected from the truncation will in turn be reflected back into the wedge. The initial reflection is however the dominant factor with respect to the damping performance of the wedge. Therefore, the greater the Martensite formation the less effective the welded wedge would be.

The TIG-welded wedge still shows significant reductions in the magnitude of the resonance peaks. Theoretically this is expected as the wave should propagate more efficiently and with less reflection through this type of weld as there is very little weld material between the wedge and strip.

The MIG-welded sample appears to have been more adversely affected by the welding process than the TIG-welded sample, and as a result, its reflection coefficient is greater, reflecting a greater proportion of the waves passing through the weld. It is quite possible that, due to the shape of the MIG weld, the materials towards the points at the centre of the weld have been affected to a greater extent than the straight edges of the TIG weld. This could result in higher levels of reflection from the Martensite area. To reduce the reflection coefficient at the two boundaries, materials with similar velocities and mass densities should be used for the filler and wedge.

In the case of the glued sample, the glue was pasted thinly and evenly between the wedge and the strip, ensuring a uniformed bond with little to no air gaps. Although the glue creates a greater impedance change in the bond than that of the welds, the glue layer itself is very thin. Moreover, the metal surrounding the bond is not exposed to the thermal cycles induced by weld formation, and therefore not affected by the heat defects that are associated with welded bonds. There are however limited practical applications for a glued bond as it is weak and can be easily broken.

6 Conclusions

In the present work, the effects of deviations of real manufactured wedge-like structures from ideal elastic wedges of power-law profile on damping flexural vibrations have been investigated experimentally. In particular, the effect of mechanical damage to wedge tips has been investigated, including tip curling and early truncation, as well as the placement of absorbing layers on different wedge surfaces. Also, the effects of welded and glued bonding of wedge attachments to basic rectangular plates (strips) have been studied.

It has been demonstrated that the effect of tip damage (curling) in a wedge of the maximum possible (extended) length allowed by manufacturing is not detrimental for its performance when compared to the same wedge that has been cut to a reduced length (truncated) in order to avoid curling. Despite the damage to the extended wedge tip, the increased length and resultant decrease in tip thickness provided the most efficient damping of flexural vibrations. The longer and thinner the wedge tip the greater the contribution of the acoustic black hole effect into the overall vibration damping, in spite of the resulting technological damage to the tip. Despite this increased damping performance, the practical applications of an extended tip are limited though due to an increased possibility of the tip braking off because of its increased fragility, although this can in part be countered by the addition of a damping layer.

It has been shown that the position of the damping layer in relation to which surface it is attached does make difference to the level of damping achieved, contrary to the predictions following from the geometrical-acoustics theory applied to idealised wedges. The measured results show that a greater damping effect occurs when the damping layer is placed on the underside flat surface of the wedge. The most likely reason for that is that due to the stepped

nature of the top of the profile the damping layer does not make full contact with the entire surface beneath the strip, thus resulting in the reduced damping effect.

It has been demonstrated that attaching power-low profiled wedges to a rectangular plate (strip) by welding or via glue results in damping performances that generally isn't any worse than the performance of a homogeneous sample containing the same wedge.

In particular, the experiments show that the glued wedge performs better than the TIG-welded wedge at frequencies below 3.7 kHz, while the TIG-welded wedge is performing better than the glued wedge over the remaining frequency range 3.7 –9 kHz. Out of the three bonds tested, the TIG-welded wedge proved to yield the most significant reductions in the amplitude of the reference strip resonance peaks over the largest frequency range.

In all four samples, the TIG-welded, MIG-welded, glued and homogeneous sample, the increased extended length results in an overall increased damping effect despite significant machining damage to the extended tip. The tears and small holes also result in enhanced damping performance due to increased scattering.

The main conclusion that can be drawn from the present experimental work is as follows. Although the above-mentioned geometrical and material imperfections generally reduce the damping efficiency to various degrees, the method of damping structural vibrations using the acoustic black hole effect is robust enough and can be used widely without the need of high precision manufacturing.

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Figure captions

Fig. 1. Wedge of power-law profile.

Fig. 2. Machine damage to a wedge tip.

Fig. 3. Bonded wedge samples: (a) –TIG-welded wedge, (b) –MIG-welded wedge, (c) - glued wedge (standard 'super glue'), (d) - homogeneous wedge.

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Table 2. Sample dimensions.

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Table 4. Peak amplitude reduction for five glued samples containing a wedge of power-law profile, in comparison with the reference plate.

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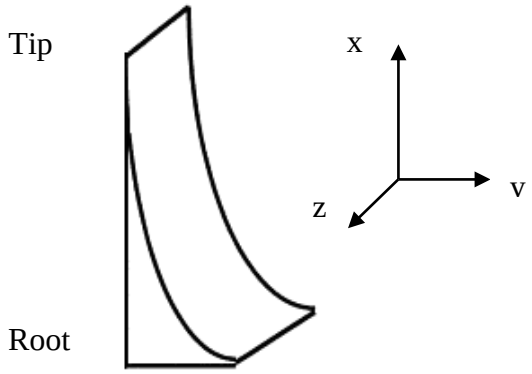

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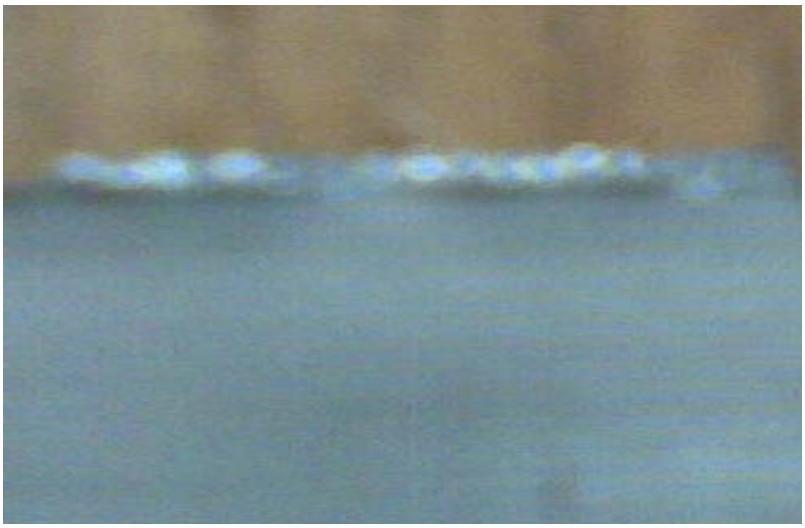

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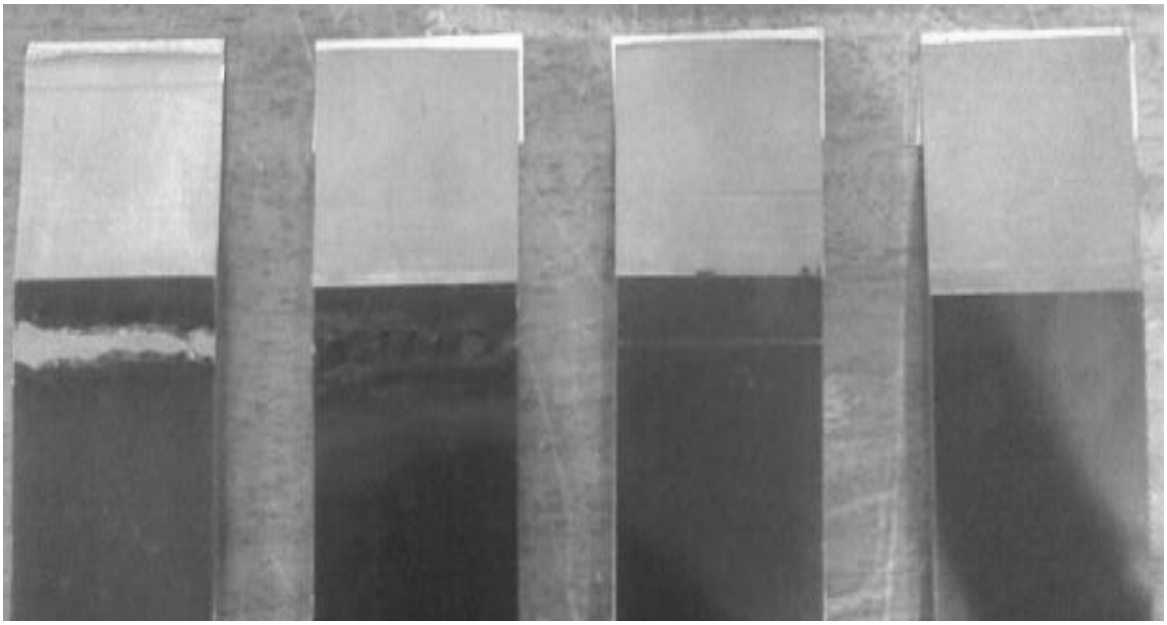

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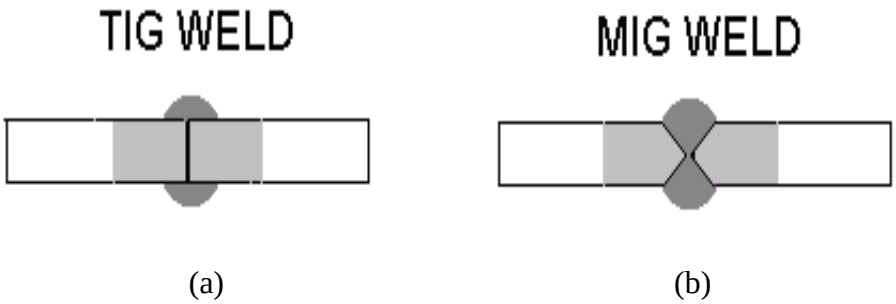

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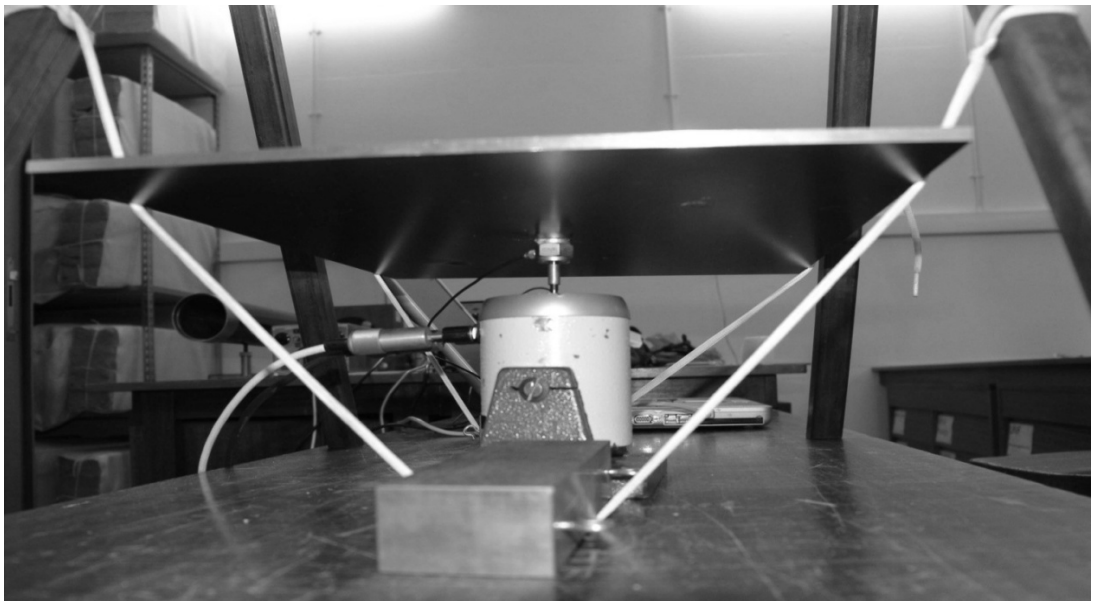

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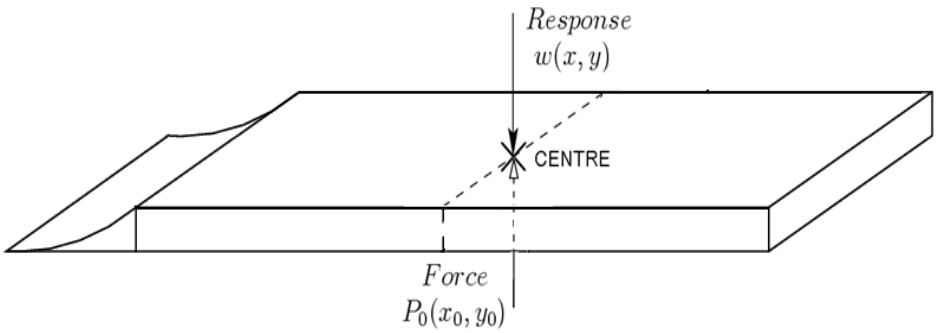

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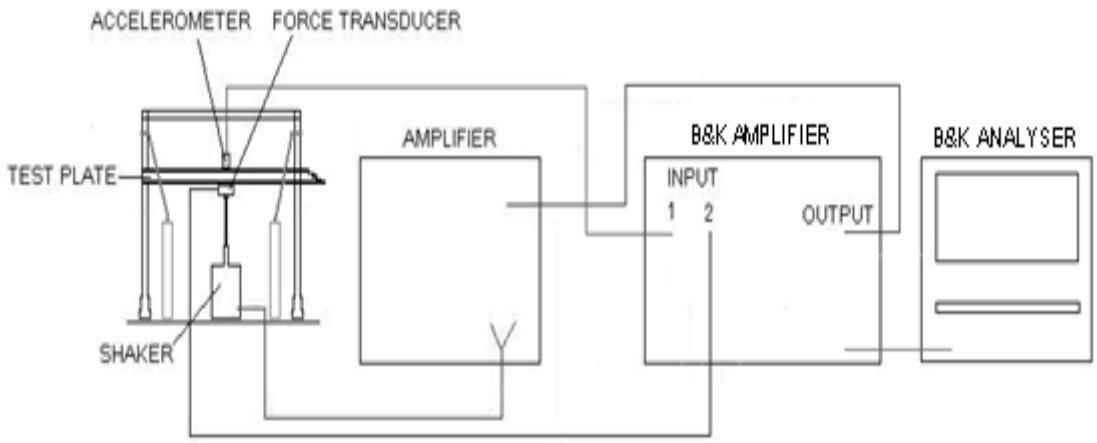

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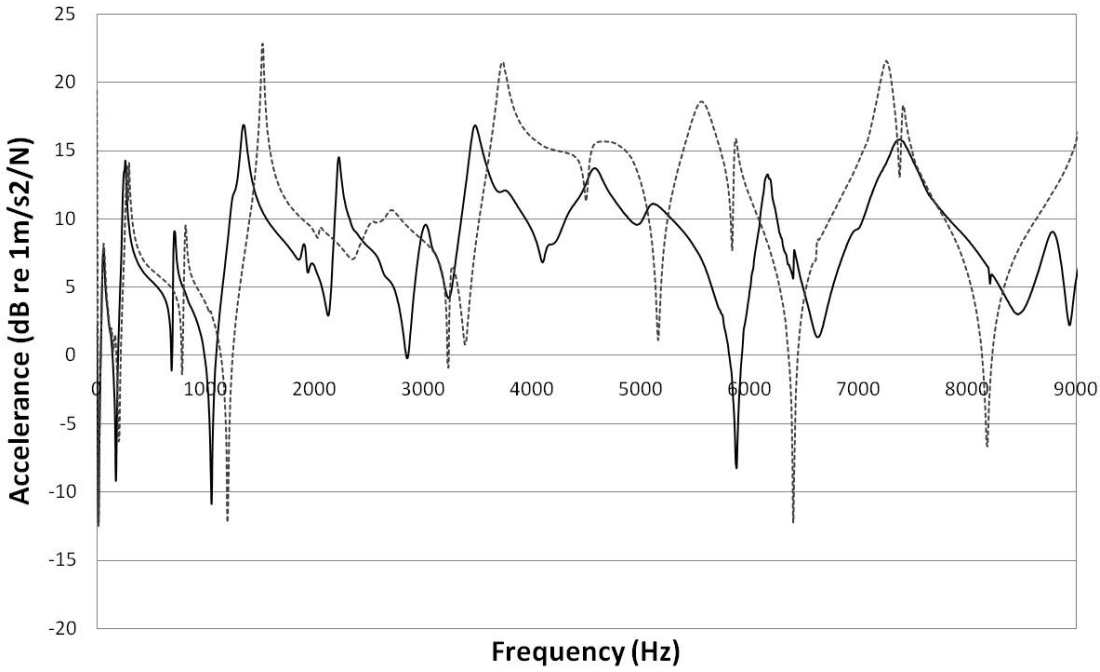

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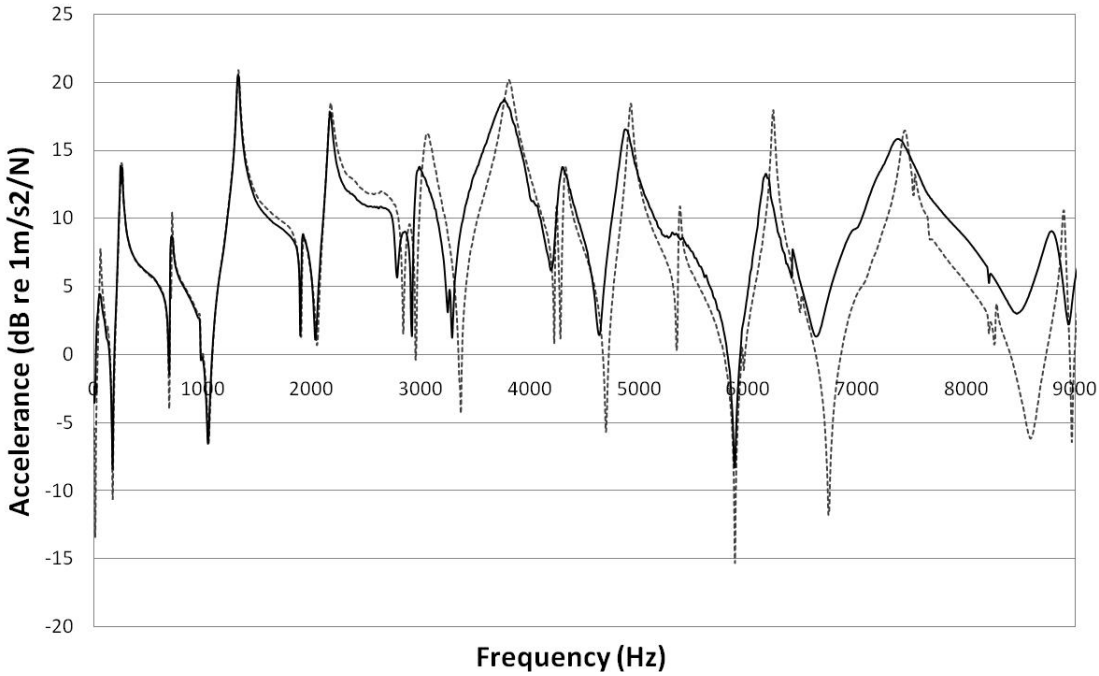

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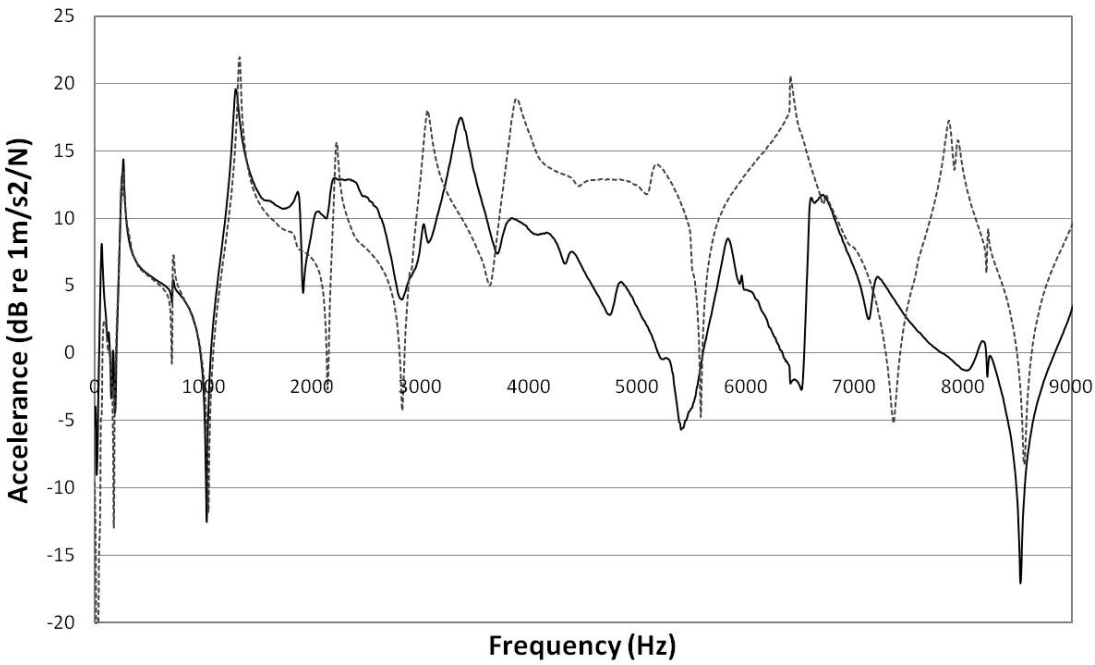

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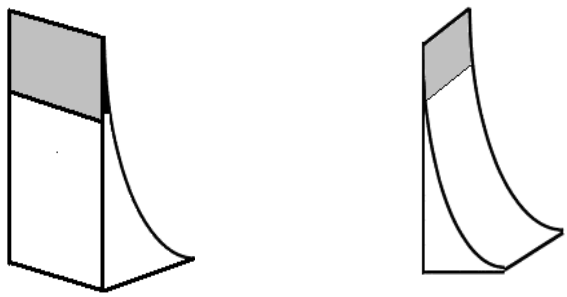

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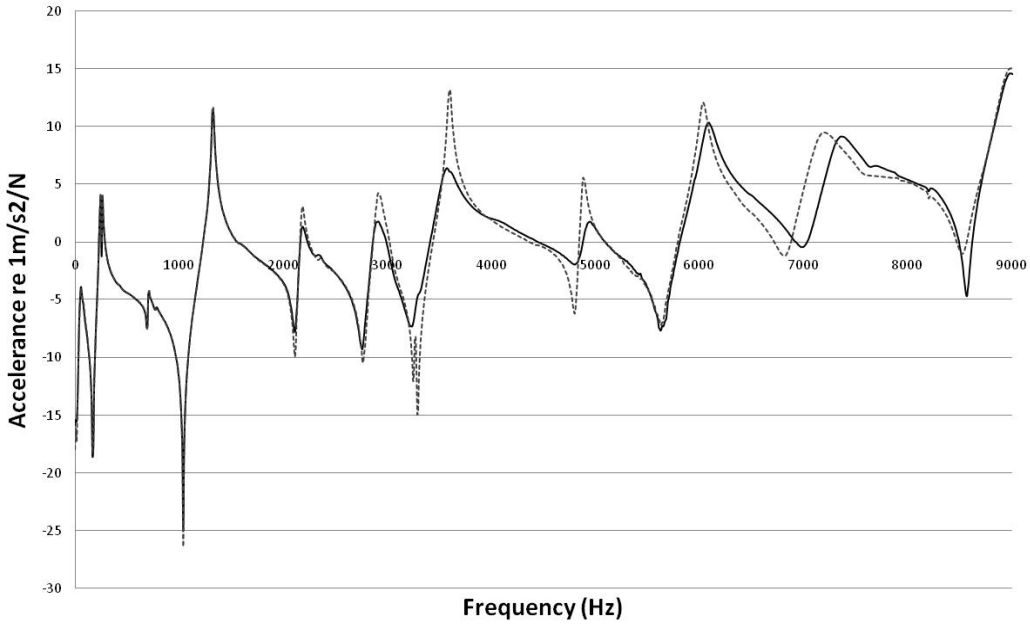

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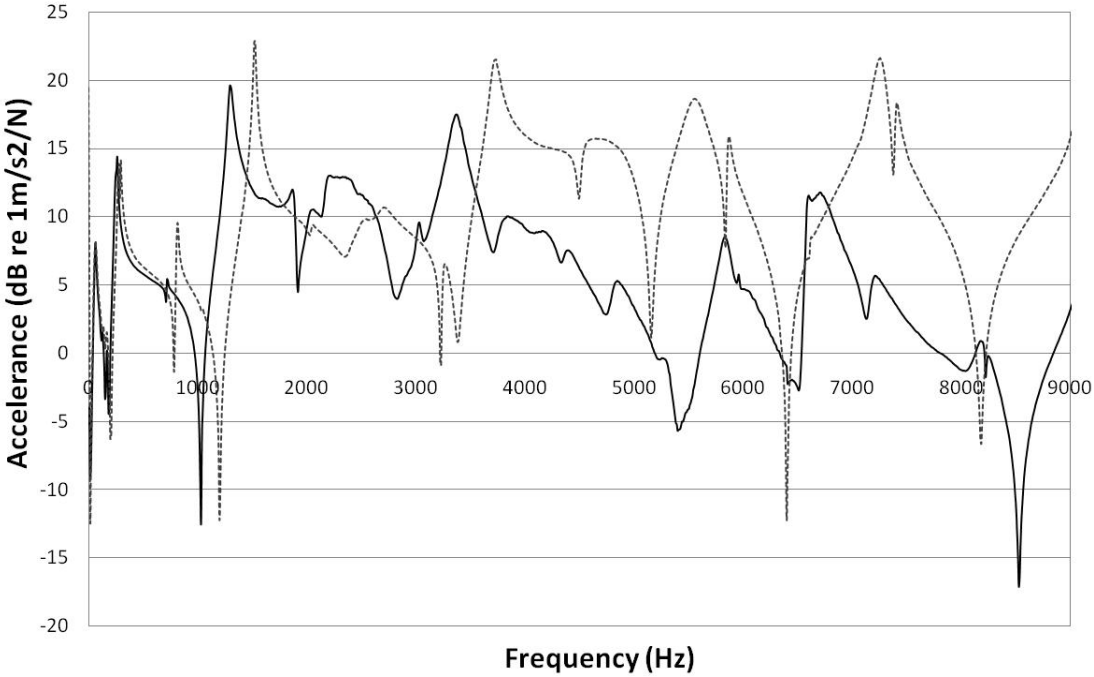

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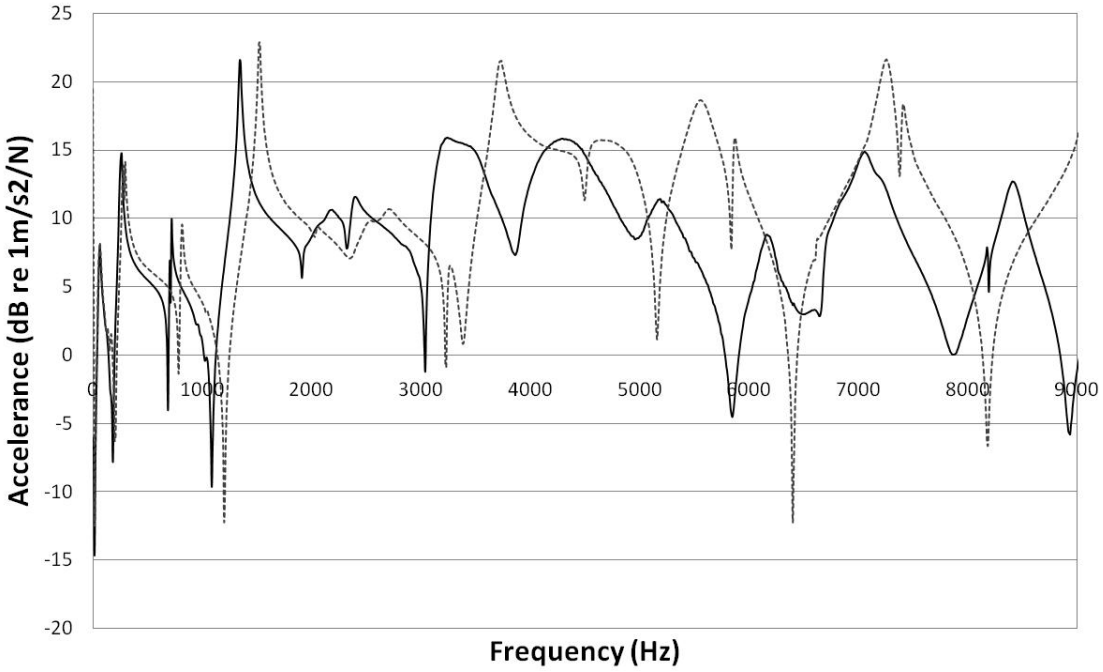

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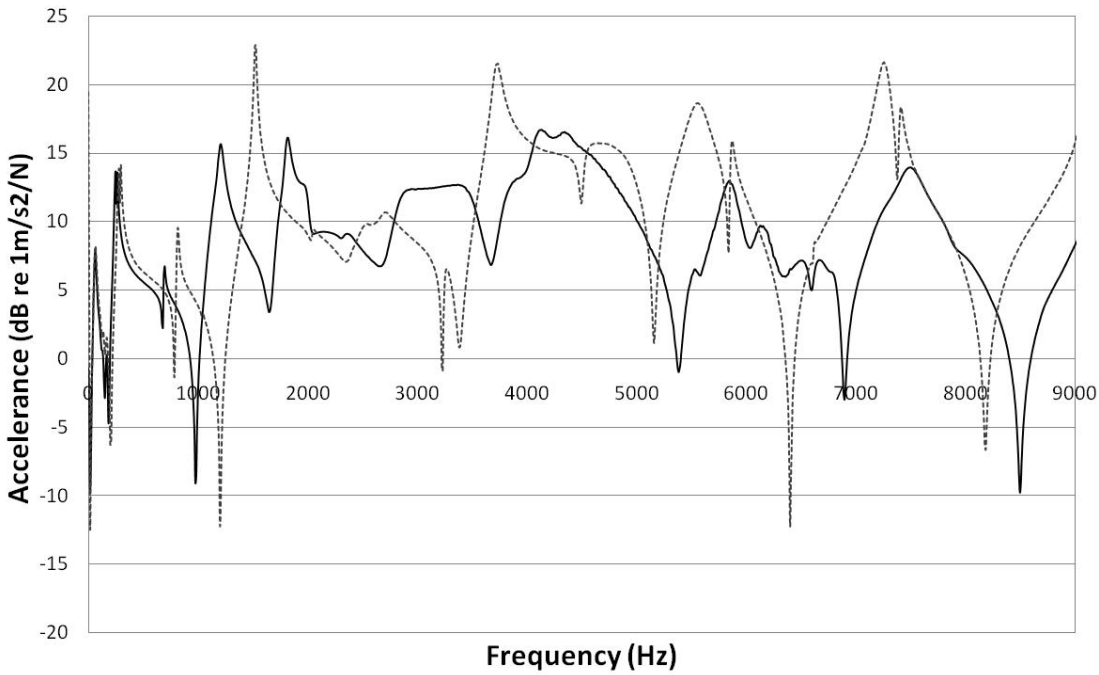
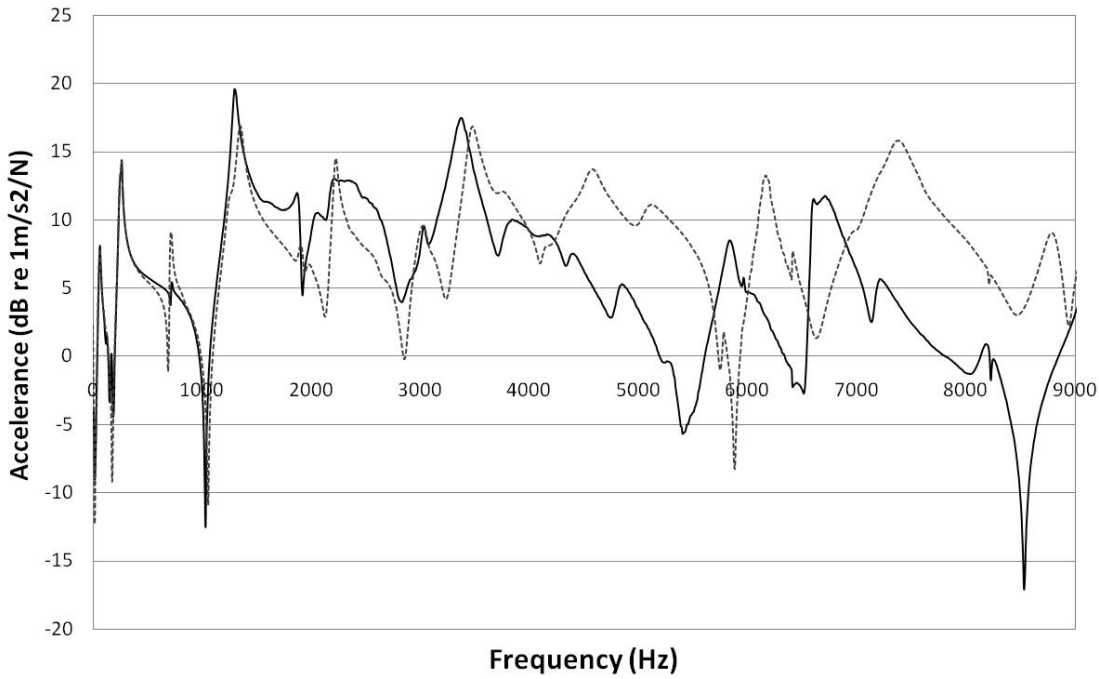


Fig. 15. Measured accelerance for a sample with the glued power-law profile (m = 2.2) (solid curve) compared to the reference plate (dashed curve).



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Tables

	Thickness	**Young's modulus**	**Density**	**Poisons ratio**	**Loss factor**
Plate	5.04 mm	190 GPa	7000 kg/m^3	0.3	0.6 %
Damping tape	0.08 mm	-	300 kg/m^3	-	6 %

Table 1. Material properties of plates and damping layers.

	Reference plate (strip)	**Extended samples**	**Homogeneous sample**	**Bonded samples**
Length	300mm	349mm	339mm	339mm
Width	50mm	50mm	50mm	50mm

Table 2. Sample dimensions.

Sample		Homogeneous				
Peak Frequency (kHz)		2	3.5	5	6	7.2
1	Peak Reduction (dB)	1.87	4.56	2.72	1.44	0.46
2	Peak Reduction (dB)	1.78	4.32	2.61	1.34	0.32
3	Peak Reduction (dB)	2.05	4.95	3.07	1.33	0.82
4	Peak Reduction (dB)	1.85	4.68	2.57	1.29	0.37
-	**Average Reduction (dB)**	1.89	4.63	2.74	1.35	0.49

Table 3. Peak amplitude reduction due to the repeated attachment of pieces of damping tape to the flat side of a homogeneous wedge, in comparison with their attachment to the profiled side.

Sample		**Glued**			
Peak Frequency (kHz)		1.8	4	5.5	7.2
1	Peak Reduction (dB)	7.5	5.5	5	7
2	Peak Reduction (dB)	7.2	5.6	5.2	7.1
3	Peak Reduction (dB)	6	4.8	4.2	6.6
4	Peak Reduction (dB)	6.9	4.6	4.4	6.3
5	Peak Reduction (dB)	7.2	5.4	5.3	7.3

Table 4. Peak amplitude reduction for five glued samples containing a wedge of power-law profile, in comparison with the reference plate.