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# Experimental investigation of damping flexural vibrations in plates containing tapered indentations of power-law profile

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## Abstract

In the present paper, experimental results are reported on damping flexural vibrations in rectangular plates containing tapered indentations (pits) of power-law profile, with the centres of the indentations covered by a small amount of absorbing material. In the case of quadratic or higher-order profiles, such indentations materialise two-dimensional acoustic 'black holes' for flexural waves that can absorb almost 100 % of the incident elastic energy. In the present

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investigation, pits have been made in different locations of rectangular plates, and the corresponding frequency response functions have been measured. It has been found that basic power-law indentations, with no or very small central hole, result in rather low reduction in resonant peak amplitudes, which may be due to the relatively small effective absorption area in this case. To increase the damping efficiency of power-law profiled indentations, this absorption area has been enlarged by increasing the size of the central hole in the pit, while keeping the edges sharp. As expected, such pits, being in fact curved power-law wedges, result in substantially increased damping. When multiple indentations are used, the resultant damping increases substantially, as expected, and may become comparable if not greater than that achieved by one-dimensional wedges of power-law profile.

*Keywords:* Vibration damping; Acoustic black hole effect; Wedges of power-law profile; Circular indentations.

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## **1 Introduction**

Passive damping of structural vibrations is usually achieved by adding layers of highly absorbing materials to the structure in order to increase energy dissipation of propagating (mostly flexural) waves [1-3]. Another well-known approach to suppression of resonant vibrations of different structures is to reduce reflections of structural waves from their free edges [4].

To implement the latter approach in a more efficient way, a new method of damping flexural vibrations based on the so-called “acoustic black hole effect” has been recently

developed and investigated [5-7]. This method has been initially applied to one-dimensional plates of power-law profile (wedges) that had to be covered by narrow strips of absorbing layers near sharp edges. The equation of the power-law is given by  $h(x) = \varepsilon x^m$ , where  $h(x)$  is the local thickness of the profile, and  $\varepsilon$  and  $m$  are positive constants. Ideally, if the power-law exponent  $m$  is equal or larger than two, the flexural wave never reaches the sharp edge and therefore never reflects back [5-8], which constitutes the acoustic black hole effect. It has been established theoretically [5,6] and confirmed experimentally [7] that this method of damping structural vibrations is very efficient even in the presence of edge truncations and other imperfections. This has been demonstrated also by numerical calculations combined with experimental testing for rectangular plates with attached power-law-shaped wedges [9] and by experimental investigations of the effects of geometrical and material imperfections [10]. Note that traditional methods of vibration damping in plate-like structures involve the use of significant quantities of viscoelastic material on the plate surface, which is not very efficient and adds significant mass to the plate. The key advantage of the black hole effect is a significant reduction in added mass required, as only the edge of the tapering wedge has a damping layer applied.

The focus of the present work is on the experimental investigation of damping flexural vibrations in plates containing two-dimensional tapered indentations (pits) of power-law profile, with the addition of a small amount of absorbing material. In the case of quadratic or higher-order profiles, such pits materialise two-dimensional ‘black holes’ for flexural waves. To understand basic principles of operation of such ‘black holes’, a geometrical acoustics approach to analysing flexural wave interaction with power-law indentations has been developed [11]. This approach shows that, if a flexural wave propagates towards a profiled indentation, some of the incident rays become captured and their reflection from the free edge

of a small internal hole in the pit can be calculated in the same way as in the case of one-dimensional wedges.

The first experimental investigation of two-dimensional acoustic black holes has been described in the paper [12] dealing with flexural vibration damping in elliptical plates. It has been demonstrated in this paper that such indentations can be very efficient dampers if they are placed in one of the plate's foci, when a shaker exciting flexural vibrations is applied to another focus. In this case, the whole energy of converging flexural waves is focused at the centre of the indentation and the problem can be described by a one-dimensional theoretical model which is based on a tapered beam [13, 14]. In the papers [15, 16], circular indentations of power-law profile have been placed in the centre of a circular plate. It has been shown both theoretically and experimentally that such indentations also act as vibration dampers, albeit not as efficient as in the above mentioned case [12-14] utilising focusing of flexural waves in elliptical plates. Nevertheless, the reduction in vibration amplitude for a cylindrical plate with a central damped indentation of quadratic profile still was essentially larger than for a homogeneous circular plate of the same dimensions completely covered with the same damping material.

In the present paper, we describe experimental investigations of damping flexural vibrations in rectangular plates containing circular indentations of power-law profile placed in arbitrary locations (see Fig. 1). Using circular indentations inside plates offers a range of benefits in comparison with wedges of power-law profile attached to plate edges. First of all, the potentially dangerous sharp edges of power-law wedges are eliminated, so that all exposed sides of the plate are of the same nominal thickness, which brings a safety benefit. Secondly, two-dimensional pits can be applied to suppress just some selected resonant peaks, when placed in certain positions. Some of the issues considered in this paper have been discussed at the international conference ISMA 2010 [17]. However, the experimental plates

used in [17] contained quadratic indentations ( $m = 2$ ), whereas in the present paper all indentations are of the fourth-power profile ( $m = 4$ ).

In the following sections of this paper, the manufacturing method used to produce the experimental plates with two-dimensional power-law indentations is described, followed by the description of the experimental set-up. The results of the measurements are then discussed, followed by the conclusions.

It is demonstrated in this paper that basic power-law pits that are just protruding over the opposite plate surface cause rather small reduction in resonant peak amplitudes, which may be due to their relatively small absorption cross-section capturing a relatively small number of incident flexural wave rays. Note that for elliptical plates this disadvantage has been overcome by focusing of flexural waves in the pit [12-14].

To increase the damping efficiency of power-law profiled indentations, the absorption cross-section has been enlarged by increasing the size of the central hole in the plate while keeping the edges sharp. As expected, such large pits, being in fact curved power-law wedges, result in substantially increased damping. However, to achieve or exceed the damping provided by one-dimensional wedges of power-law profile, multiple circular indentations must be made within a plate.

## **2 Manufacturing of experimental samples**

All experimental samples in the present work were manufactured from 5 mm thick hot drawn mild steel sheets; which are more resistant to mechanical stresses incurred in the manufacturing process than cold drawn steel sheets, thus resulting in fewer internal defects.

Dimensions of the produced rectangular plates were 400 x 300mm. Approximate material properties of plates and visco-elastic damping layers are listed in Table 1.

A CNC (Computer Numerically Controlled) milling machine operating at a cutter speed of 1200 rpm was used to produce circular fourth-power indentations into the plates. Three types of experimental samples were produced for this investigation, a plate with a singular circular indentation (Fig. 2), a plate with a singular circular indentation with a drilled central hole (Fig. 3(a)), and plates containing multiple profiled circular indentations with central holes (Fig. 3(b)).

There are three main problems encountered when utilizing this method of manufacturing. The first being that at the centre of the indentation, where the material thickness is less than 0.4 mm and the machining stress and resulting heat are high, there may be blistering (see Fig. 4(a)) leading to inaccurate results during experimental testing. Secondly, it is the formation of a machine line, as the cutters movement through the indentation is computer controlled. It merely moves from one programmed height to another, gouging the material and creating a raised line in the indentation, which, as with blistering, could lead to increased elastic wave scattering at higher excitation frequencies. Finally, additional damage can occur when a hole is drilled into the centre of the circular indentation. Due to the small thickness of the material at this point, it is more susceptible to tearing.

### **3 Experimental setup**

The experimental set-up has been designed to allow nearly free vibration of the sample plates (i.e. to eliminate clamping of edges), take the weight off the plate edges and introduce minimal damping to the system, see Fig. 5.

The excitation force was applied to the centre of the plate via an electromagnetic shaker attached to the plate using ‘glue’ and fed via a broadband signal amplifier. The response was recorded by an accelerometer (B&K Type 4371) that was attached to the upper surface, directly above the force transducer (B&K Type 8200), also via ‘glue’ (see Fig. 6). The acquisition of the point acceleration was utilised using a Bruel & Kjaer 2035 analyser and amplifier over a frequency range of 0-9 kHz, a schematic is shown in Fig. 7. The results presented in this paper have been averaged over five measurements in order to ensure a statistically accurate representation of the point acceleration.

Two different styles of reference plate can be considered for comparison: a plain plate and a plate with a punched through hole [17]. Both reference plates are of the same dimensions as the profiled plates, the punched hole plate contains a through cylindrical hole of the same diameter and position as the machined power-law profiles. The plain plate more accurately represents a practical situation, with the profile being cut into an existing structure. It also has reduced internal defects resulting from machining stress, when compared to the punched hole plate, although the latter accounts for the reduction in mass and equivalent stiffness due to the profiled indentation. Keeping the above in mind, a plain reference plate was used for comparison in this paper.

## **4 Experimental results and discussion**

### *4.1 A profiled circular indentation with and without a small central hole*

Two sets of experimental results are described in this section: the effect of adding a circular indentation of power-law profile into a plate, when compared to a reference plate, and the effect of drilling a 2mm hole in the centre of the indentation.

Our initial measurements with a single circular indentation without a central hole demonstrated that in this case there were no noticeable damping effects when compared to a reference plate, apparently because of a rather thick central area due to limitations of the manufacturing method used. There was little to no improvement in damping by the addition of a damping layer. To the contrary, in some frequency ranges the presence of the indentation actually increased the level of vibrations in the plate. From these initial observations it has been decided that a central hole has to be drilled into the centre of the indentation.

Note that due to manufacturing limitations at the centre of the circular pit, there is an area of almost equal thickness of about 0.4 mm that extends from the central point out to a diameter of approximately 3 mm. A hole of 2mm diameter can therefore be drilled without affecting the minimum tip thickness (Fig. 4(b)).

Figure 8 shows the measured accelerance for an indentation of profile with a 2 mm central hole with and without a damping layer compared to a reference plate. When comparing an indentation with a 2mm central hole and damping layer to a reference plate it can be seen, below 4.7 kHz there is little to no damping. A slight increase is seen in some of the peaks between 1.5 – 3 kHz and 5.5 -6 kHz. A maximum damping of 6 dB occurs at 6.5 kHz. These results show that the effect of drilling a hole in the centre of the circular inclusion increases the overall losses at higher frequencies, most likely due to increased absorption of the incident flexural wave.

These measurements, however, also show some specific behavior as a result of adding a damping layer to the tip of the circular inclusion. Earlier, it has been shown for power-law wedges [7] that in order to achieve significant damping there is a requirement to add an

additional damping layer to the tip of the wedge. However, this is not the clear case for a circular indentation with a 2 mm central hole, with only a 1-2 dB additional reduction occurring. Damping obtained by the addition of a damping layer is dependent on the stiffness of the damping layer itself and whether the damping layer also contained a drilled hole.

#### 4.2 *A profiled circular indentation with a large central hole*

In attempts to improve the damping efficiency of the profiled indentations, the central hole size was increased progressively by 2 mm until a central hole size of 14 mm and an indentation diameter of 100 mm was produced. As the central hole size increased so did the damping performance of the circular indentation. This is most likely due to the increased boundary area of the tapered surface and the indentation more closely resembling a wrapped around wedge. As the central hole increases so does the diameter of the indentation.

Figure 9 shows a direct comparison between the profiled circular indentations with 2 mm, 10 mm and 14 mm central holes. Below 3.5 kHz, the response of all three samples are almost identical, varying by no more than approximately 1-2 dB. In all other regions the 14 mm central hole has an increased damping performance compared to the 2 mm and 10 mm central holes. In the region 6-8 kHz, the maximum difference can be seen with the response of the 14 mm central hole. It shows a reduction by a maximum of 2.5 dB compared to the sample containing the 10 mm central hole and by 6 dB compared to the sample containing a 2 mm central hole. Increasing the central hole diameter increases damping performance of the circular indentation as it increasingly resembles a curved wedge as the diameter of the central hole is enlarged and the curvature of the free edge diminishes.

Unlike in the case of the indentation containing a 2 mm central hole, the addition of a damping layer increases the damping performance of the inclusion up until 8 kHz, after which it has a reduced effect. This pattern of varying damping is consistent with the theory of power-law profiled wedge damping [5-7]. The greatest increase in damping performance was achieved in the frequency range between 3.5- 8 kHz, when an additional damping layer was attached.

A comparison of the results for a profiled circular indentation with a 14 mm central hole and an additional damping layer to the results for a reference plate are shown in Fig. 10. Again, Fig. 10 shows that below 2 kHz the circular pit provides little to no damping. In the region of 3.8 – 9 kHz, damping varies between 1 – 10 dB, and maximum damping occurs at 6.5 kHz.

#### *4.3 Damping effect of increasing the diameter of the circular inclusion*

This section describes the effect of increasing the diameter of the circular indentation from 100 mm to 114 mm. By increasing the size of the indentation, the length of the profiled area on the plate increases. The two samples tested had a hole of 14 mm in the centre of the indentation. The results are shown in Fig. 11.

From Fig. 11, it can be seen that increasing the diameter of the indentation increases the damping performance. Below 2.5 kHz there is little to no discernable difference between the two samples, as the flexural wavelength is longer than the diameter of the inner aperture of the smaller diameter hole. Above 2.5 kHz the damping performance of the larger diameter hole increases compared to the smaller diameter hole. A maximum of a 5 dB increase in damping occurs with the 114 mm diameter indentation at 4 kHz.

#### 4.4 *Multiple circular indentations*

In the previous section, the comparison of a reference plate was made with a similar rectangular plate with a damped circular indentation of 14 mm diameter, leading to a reduction in amplitude of up to 10 dB at 6.5 kHz. In this section, the results of increasing the number of damped circular indentations on a centrally excited plate for two, four and six profiles are presented, when compared to each other and to a reference plate. Naturally, the multiple-indentations samples were expected to perform better than the plates with one circular indentation, and the damping performance was expected to increase with the number of circular indentations due to a cumulative effect. The diameters of the circular indentations were 114 mm and the central holes were 14 mm in diameter.

Figure 12 shows the results for the plates containing two, four and six profiled circular indentations with 14 mm central holes and additional damping layers. It is seen that the greater the number of indentations in the plate, the greater the damping obtained. The frequency at which a substantial reduction in peak amplitude is first seen decreases as the number of indentations increases. For the two, four and six indentation plates this occurs at approximately 4, 3 and 2 kHz respectively, as the lower frequency mode shapes start to correspond more closely with parts of the indentations patterns. After 4 kHz, it can be seen that the response clearly flattens increasingly with indentation number, thus increasing the level of broad band damping achieved compared to the high level of damping perceived at a specific frequency seen in the one indentation plate.

The results for a plate containing six profiled circular indentations with 14 mm central holes and additional damping layers, compared to a reference plate, are shown in Fig. 13. This configuration by far has the greatest damping at higher frequencies out of the samples

tested, and it even outperforms a wedge of the same power-law profile. Below 1 kHz there is little to no damping, as expected, and between 1 – 2 kHz a reduction of peak amplitude by up to 4 dB is observed, after which damping increases until almost all peak responses are flattened. A maximum damping of 14 dB occurs at 6.5 kHz.

To ensure that the damping effect shown by the plate containing six profiled circular indentation was in fact due to the presence of the indentations and not just merely due to the increase in surface area of the plate covered by the visco-elastic damping layer, a comparison was made between a plain reference plate, a reference plate with the equivalent area of attached visco-elastic damping that is present on a plate containing six circular indentations and a plate containing six circular indentations. The results are shown in Fig. 14. It is clear to see that some degree of damping is obtained through the addition of the visco-elastic damping layer alone, however it does not account for the full extent of the damping performance seen when it is combined with the circular indentations of power-law profile. For example, at approximately 5 kHz the visco-elastic damping layer sees a reduction in amplitude in the range of 3 dB compared to the reference plate, and at the same instance the plate containing six circular indentations produces a reduction in the range of 12 dB.

As expected, the damping performance of the plates containing a singular indentation is not greater than the performance of 1D wedges of the same profile. However, the multiple indentation plates were able to compete with the damping performance provided by the tapered wedges. The position of the holes is linked to performance, and different combinations may result in greater levels of damping.

## **5 Conclusions**

It has been demonstrated in this paper that basic power-law indentations with no central hole cause very small or no reduction in resonant peak amplitudes of plate vibrations, which may be due to their relatively small absorption cross-section capturing a relatively small number of flexural wave rays. Note that for elliptical plates this disadvantage has been overcome by focusing of flexural waves in the pit [12-14]. Introduction of a 2 mm central hole improved the situation and increased damping.

To increase damping even more, the absorption cross-section has been enlarged by increasing the size of the central hole in the indentation up to 14 mm, while keeping the edges sharp. As expected, such pits, being in fact curved power-law wedges, resulted in substantially increased damping performance that was comparable with that achieved by one-dimensional wedges of power-law profile. Also the larger the diameter of the indentation the greater the damping effect due to increased length of the profiled area on the plate.

As expected, the introduction of multiple-hole plates in the current layout clearly increases the damping performance of the two-dimensional indentations of power-law profile. As the number of indentations increase so does the damping performance. The six indentation sample shows an increase in resonant peak damping over a large frequency range. The obtained results show that using combinations of circular indentations of power-law profile with large central holes can be a very efficient method of damping flexural vibrations in different plate-like structures.

## **Acknowledgements**

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## Figure captions

Fig. 1. Drawing of a circular power-law indentation in a rectangular plate

Fig. 2. Plate containing a circular indentation with no central hole.

Fig. 3. Profiled circular indentations with central holes: (a) - two indentations; (b) - four indentations; (c) - six indentations.

Fig. 4. Machine damage to a circular indentation (*a*); drilling a hole in the area of constant thickness (*b*).

Fig. 5. Experimental setup.

Fig. 6. Locations of the shaker (Force) and of the accelerometer (Response) on an experimental sample.

Fig. 7. Schematic of the experimental setup utilising the Bruel & Kjaer analyser.

Fig. 8. Measured accelerance for a profiled circular indentation with a 2 mm central hole with (solid line) and without (dashed line) an additional damping layer compared to a reference plate (solid grey line).

Fig. 9. Measured accelerance for a damped circular indentation with a 14 mm central hole (solid black line) compared to a damped circular indentation with a 2 mm central hole (dashed line) and a 10 mm central hole (solid grey line).

Fig. 10. Measured accelerance for a profiled circular indentation with a 14 mm central hole with an additional damping layer (solid line) compared to a reference plate (dashed line).

Fig. 11. Measured accelerance for a damped circular indentation of diameter 114 mm (solid line) compared to a circular indentation of diameter 100 mm (dashed line), both with a 14 mm central hole.

Fig. 12. Measured accelerance for a plate containing two (dashed line), four (solid grey line) and six (solid black line) profiled circular indentations with 14 mm central holes and added damping layers.

Fig. 13. Measured accelerance for a plate containing six profiled circular indentations with 14 mm central holes and additional damping layers (solid line), as compared to a reference plate (dashed line).

Fig. 14. Measured accelerance for a plate containing six circular indentations (solid black line), as compared to a reference plate (solid grey line) and to a reference plate covered by a damping layer with the total surface area equal to that of a plate containing six circular indentations (dashed line).

## **Table captions**

Table 1. Geometrical and material properties of plates and damping layers.

**Figures**

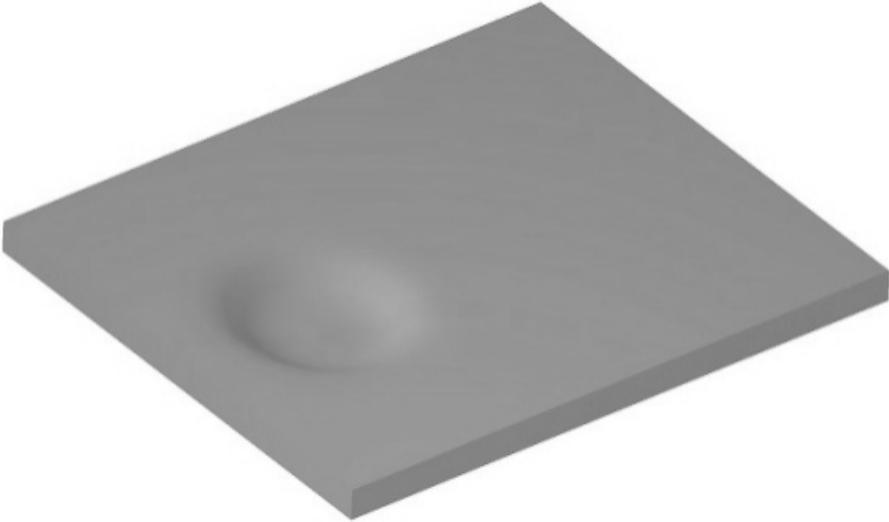


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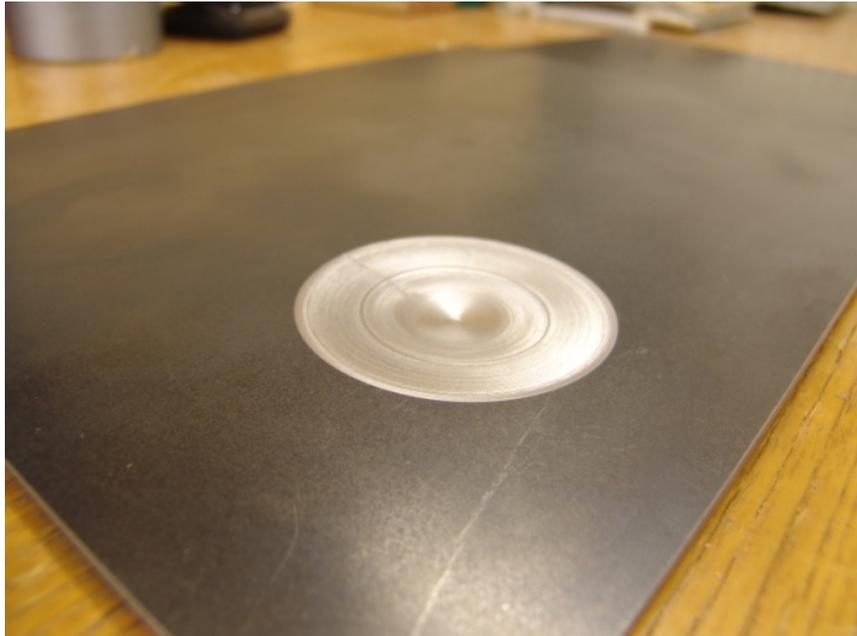


Fig. 2. Plate containing a circular indentation with no central hole.

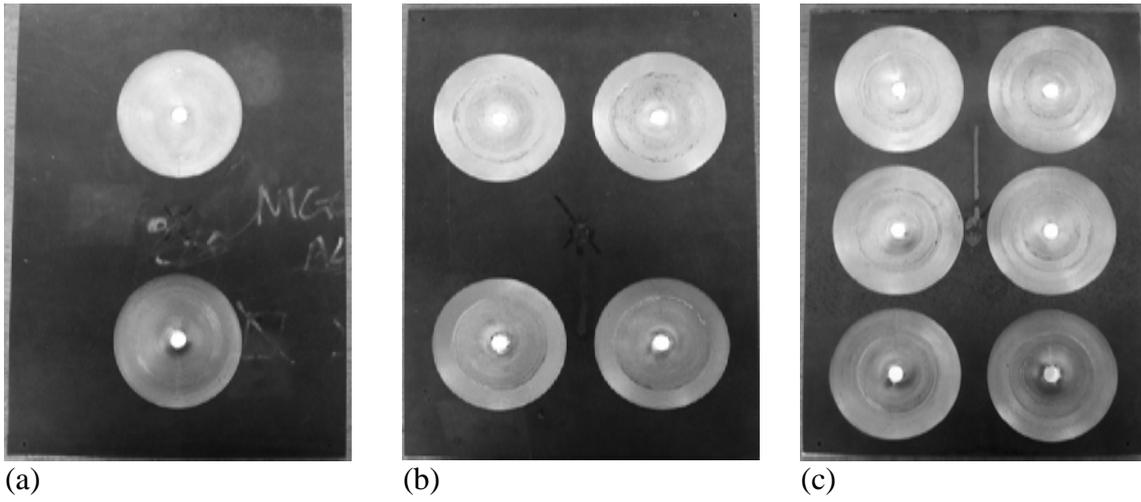
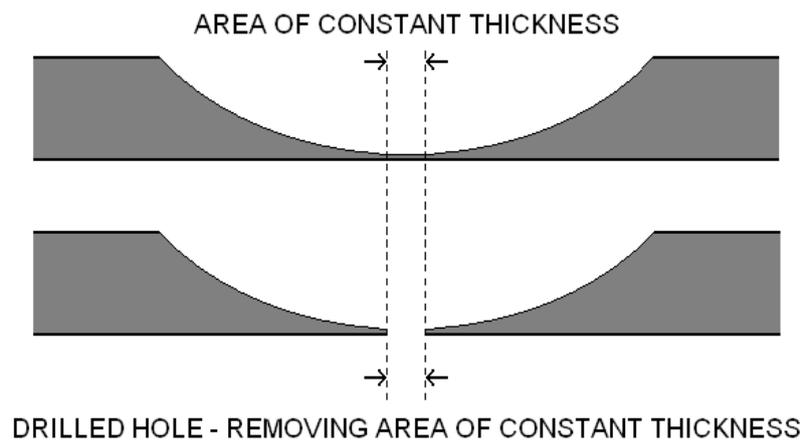


Fig. 3. Profiled circular indentations with central holes: (a) - two indentations; (b) - four indentations; (c) - six indentations.



(a)



(b)

Fig. 4. Machine damage to a circular indentation (a); drilling a hole in the area of constant thickness (b).

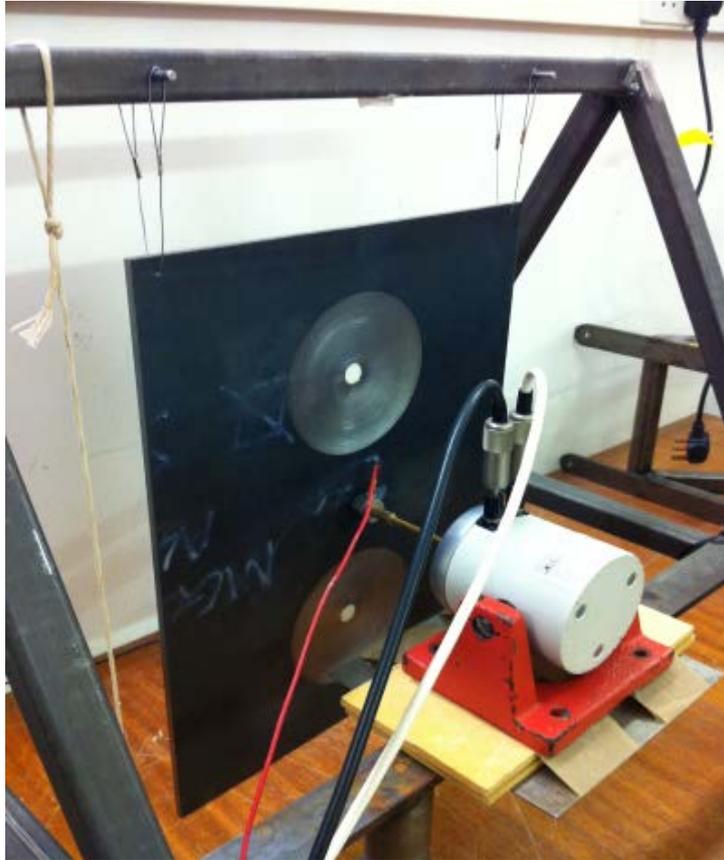


Fig. 5. Experimental setup.

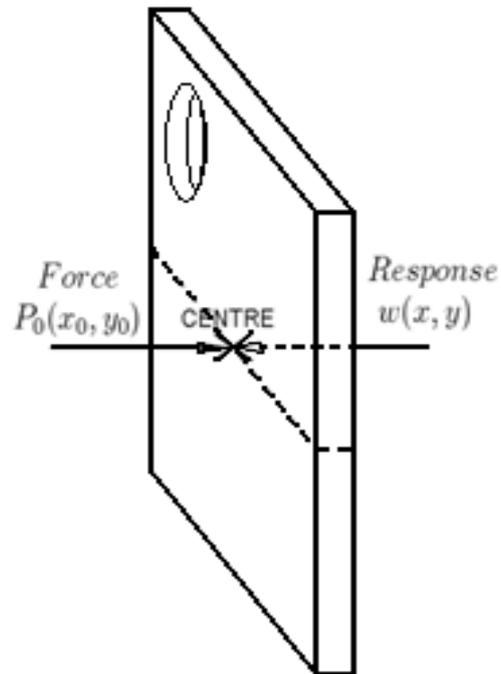


Fig. 6. Locations of the shaker (Force) and of the accelerometer (Response) on an experimental sample.

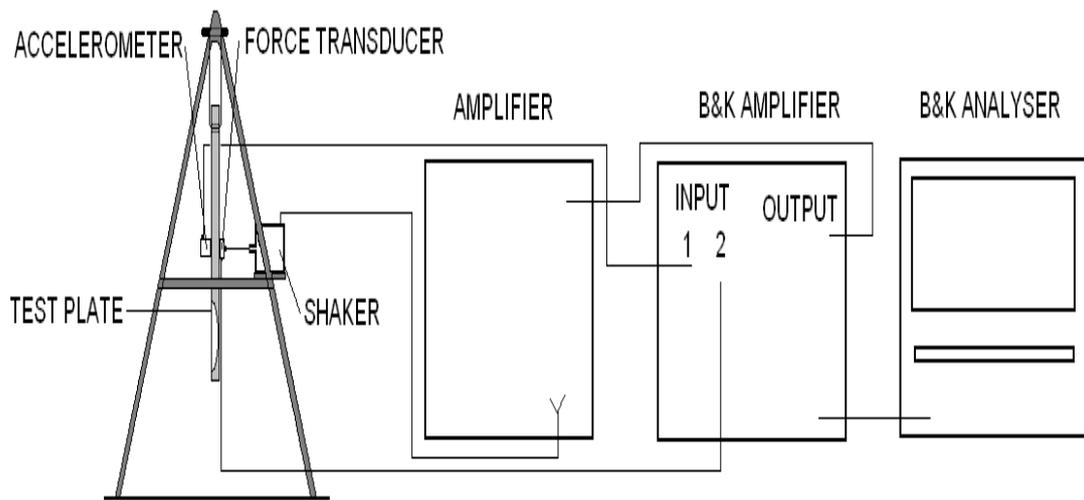


Fig. 7. Schematic of the experimental setup utilising the Bruel & Kjaer analyser.

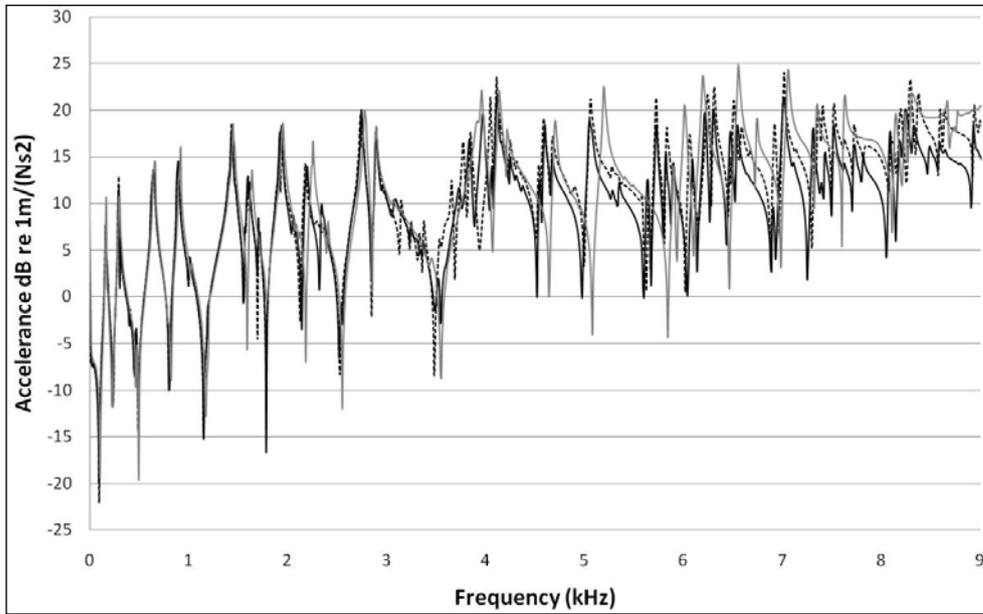


Fig. 8. Measured accelerance for a profiled circular indentation with a 2 mm central hole with (solid line) and without (dashed line) an additional damping layer compared to a reference plate (solid grey line)

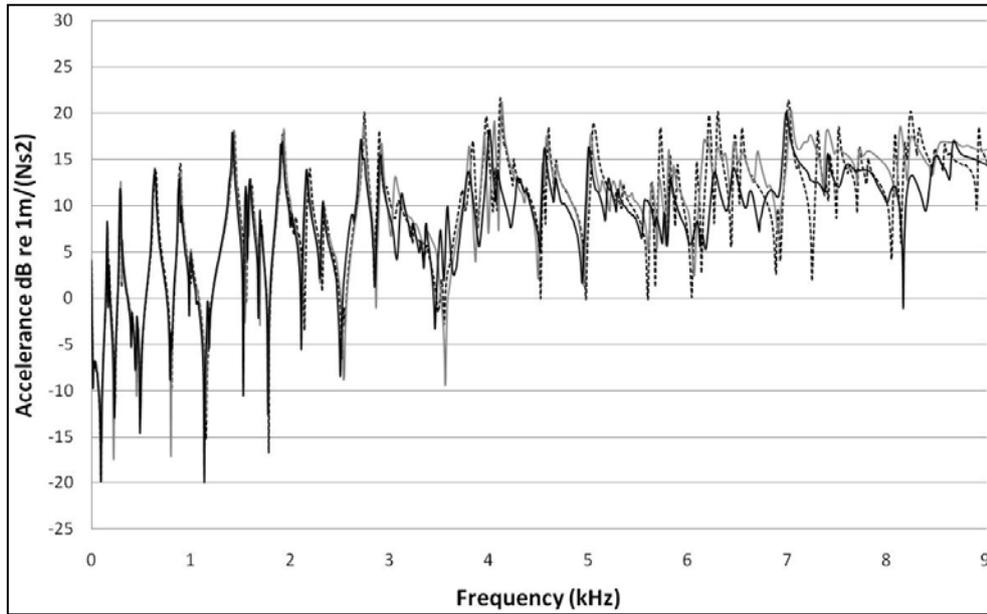


Fig. 9. Measured acceleration for a damped circular indentation with a 14 mm central hole (solid black line) compared to a damped circular indentation with a 2 mm central hole (dashed line) and a 10 mm central hole (solid grey line).

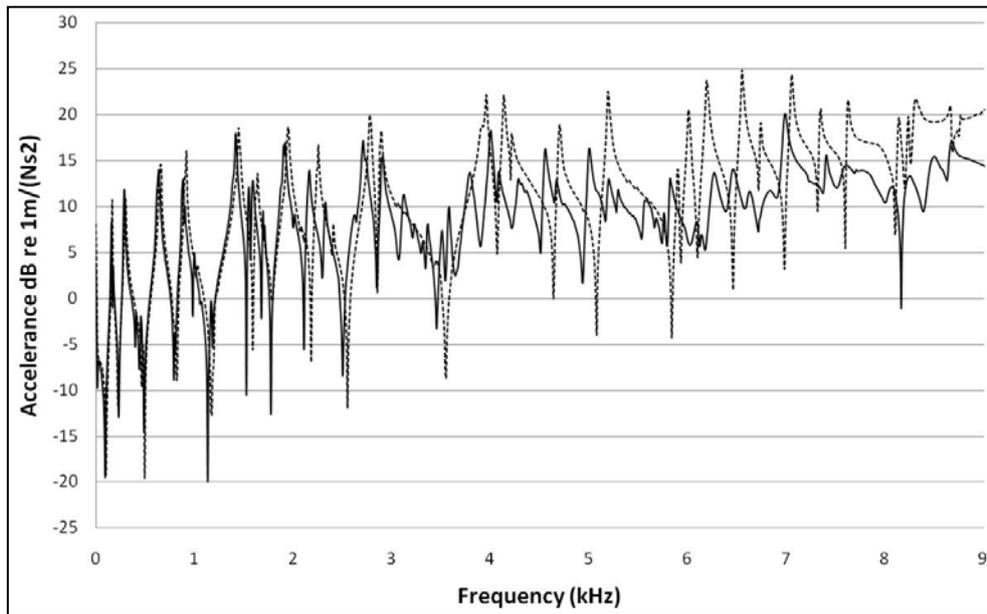


Fig. 10. Measured acceleration for a profiled circular indentation with a 14 mm central hole with an additional damping layer (solid line) compared to a reference plate (dashed line).

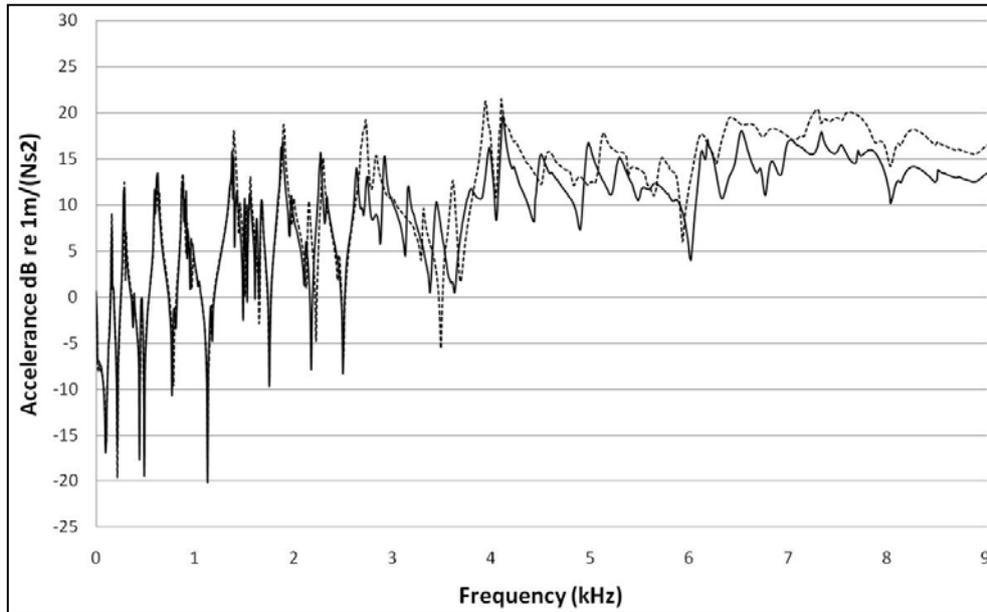


Fig. 11. Measured acceleration for a damped circular indentation of diameter 114 mm (solid line) compared to a circular indentation of diameter 100 mm (dashed line), both with a 14 mm central hole.

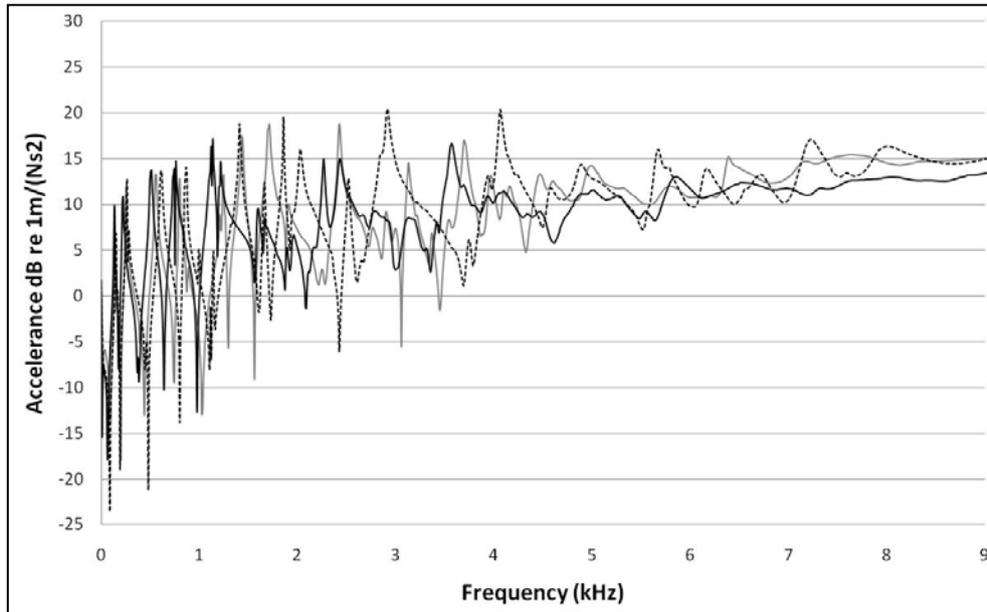


Fig. 12. Measured accelerance for a plate containing two (dashed line), four (solid grey line) and six (solid black line) profiled circular indentations with 14 mm central holes and added damping layers.

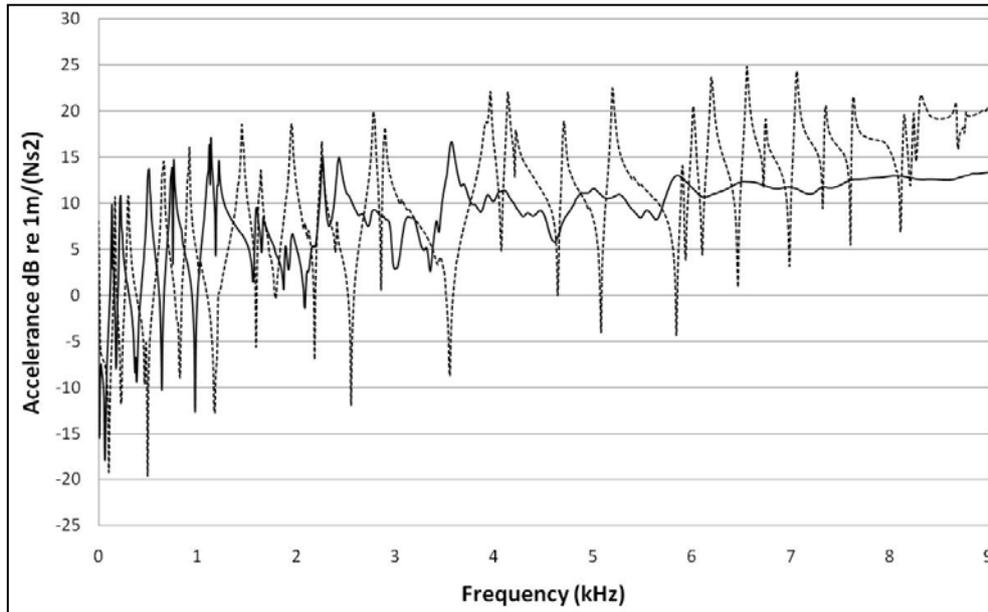


Fig. 13. Measured accelerance for a plate containing six profiled circular indentations with 14 mm central holes and additional damping layers (solid line), as compared to a reference plate (dashed line).

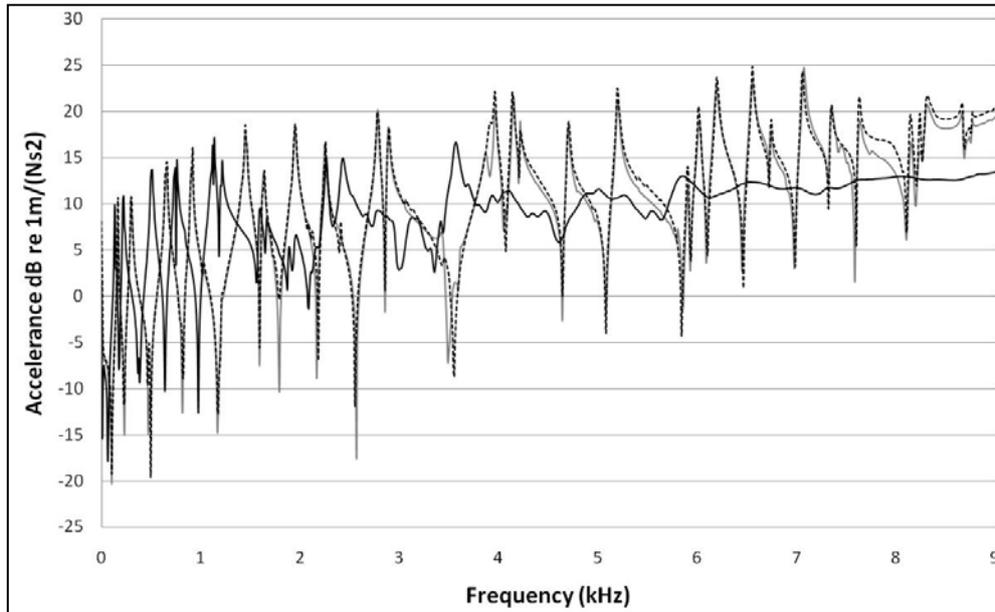


Fig. 14. Measured accelerance for a plate containing six circular indentations (solid black line), as compared to a reference plate (dashed line) and to a reference plate covered by a damping layer with the total surface area equal to that of a plate containing six circular indentations (solid grey line).

## Tables

	<b>Thickness</b>	<b>Young's modulus</b>	<b>Mass density</b>	<b>Poisson's ratio</b>	<b>Loss factor</b>
<b>Plate</b>	5.04 mm	190 GPa	7000 kg/m <sup>3</sup>	0.3	0.6 %
<b>Damping layer</b>	0.08 mm	-	300 kg/m <sup>3</sup>	-	6 %

Table 1. Geometrical and material properties of plates and damping layers.